

# DESIGNING PLATFORMS FOR CUSTOMIZABLE PRODUCTS AND PROCESSES IN MARKETS OF NON-UNIFORM DEMAND

*(short form: Designing Platforms in Markets of Non-Uniform Demand)*

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## ABSTRACT

The foremost difficulty in making the transition to mass customization is how to offer product variety affordably. The answer to this quandary lies in the successful management of modularity and commonality in the development of products and their production processes. While several platform design techniques have emerged as a means to offer modularity and commonality, they are limited by an inability to handle multiple modes of offering variety for multiple design specifications.

The Product Platform Constructal Theory Method (PPCTM) is a technique that enables a designer to develop platforms for customizable products while handling issues of multiple levels of commonality, multiple product specifications, and the inherent tradeoffs between platform extent and performance. The method is limited, however, by its inability to handle multiple design objectives and its reliance on the assumption that demand in the market is uniform for each product variant. The authors address these limitations in this paper by infusing the utility-based compromise Decision Support Problem and demand modeling techniques. The authors further augment the PPCTM by extending its use to a new domain: the design of process parameter platforms.

The augmented approach is illustrated through a tutorial example: the design of a product and a process parameter platform for the realization of a line of customizable cantilever beams.

**Keywords:** mass customization, product platforms, process parameter platforms, constructal theory

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## 1 OFFERING AFFORDABLE VARIETY THROUGH PLATFORM DESIGN

As manufacturing enterprises have struggled to meet demands for customized products through traditional economies of scale strategies, mass customization has emerged as a manufacturing paradigm for enterprises to efficiently and effectively satisfy customers' requirements for variety. Offering product variety affordably, the crux of mass customization, is the foremost difficulty that enterprises face in making the transition to this paradigm.

It is not feasible or effective to cope with customers' demands for product variety through a simple increase in inventory, a reaction commonly found in mass production. Manufacturing enterprises are recognizing that product design presents the best control over offering such variety [1]. Similarly, as a result of the shift to mass customization, the complexity of the production process design problem is dramatically increased – enterprises are forced to manufacture more complex products (multiple features, multiple variants) with reduced product life cycles, reduced time-to-market, and volatile demand [2]. As such, current manufacturing approaches must enable the quick launch of new product models, rapid adjustment of the manufacturing system capacity to market demands, and integration of new process technologies into existing systems [3].

One manner in which enterprises can efficiently handle product and production capacity variety is through the development of platforms – a set of common components, modules or parts from which a stream of variants can be created [4]. The design of platforms enables the manufacturer to maintain the economic benefits of having common parts and processes (reduced system complexity, reduced development time and costs) while still being able to offer variety to customers [5]. In this paper the authors present augmentations to an existing platform design approach (the Product Platform Constructal Theory Method) that enable a designer to systematically manage modularity and commonality in the development of both customizable products and production processes in the presence of non-uniform market demand.

## 1.1 Offering Product Variety through Platform Development

Consider the following illustrative example: a manufacturing enterprise wishes to offer a line of customizable cantilever beams. The company wants to offer product variety in multiple design specifications; specifically, they wish to provide customers the ability to specify a beam of any desired length and of any loading condition. The manufacturer and designer have decided to offer variety via three methods: (i) to change the beam cross-section, (ii) to change the beam material, and (iii) to cut the beams to customized lengths from standardized pieces. In order to efficiently offer product variety, the designer must determine the proper extent of application of these different methods of offering variety (i.e., the architecture of a product family). This decision must be made in the presence of a market with non-uniform demand (i.e., the manufacturer must produce more of some product variants than others) and two conflicting design goals: to provide the lowest average cost across the volatile market, and to provide the lowest average maximum beam deflection (essentially a quality metric) across the family.

In order to address the problem described above, a designer requires a platform design method that can consider non-uniform demand, multiple customizable specifications, multiple modes of managing product variety, and the tradeoff between commonality and product performance. At first glance, this illustrative example seems fairly simple; however, none of the various product platform design approaches that have been proposed in the literature are able to tackle all of the facets of this problem.

Bottom-up platform approaches such as Kalopakjian's Group Technology [6], Ericsson and Erixon's Modular Functional Deployment [7], and Siddique and Rosen's Product Family Reasoning System [8] provide a means for a designer to consolidate existing products to create product families. They are not appropriate for the illustrative example since product rationalization is achieved after a number of products have been designed and manufactured.

Top-down platform design methods are more relevant to this problem as they are characterized by an up-front decision to simultaneously develop a product family based on a common core and to reduce redesign cost. Examples include Nayak and coauthors' Variation-Based Platform Design Methodology [9], and Simpson and coauthors' Product Platform Concept Exploration Method [10, 11]. Such techniques provide a designer the ability to develop a product platform based upon a scaled variable and a series of commonalized design parameters.

Unfortunately, as seen in Simpson's review of 32 "optimization-based" product platform design approaches [12], there are several limitations that prevent a designer using existing top-down techniques from satisfactorily solving the seemingly simple cantilever beam example posed at the beginning of this section.

- *Synthesis of multiple techniques for managing variety for multiple design specifications:* A common limitation of existing top-down approaches is that variety is only considered in only one product specification. Typically, products are customized for multiple specifications (e.g., the torque and the power of a motor, the length and loading of a beam, etc.). Furthermore, products are customized by using multiple approaches for managing product variety (e.g., modular design, adjustable features, dimensional customization, etc.). Of those surveyed by Simpson, only two existing methods are capable of handling multiple methods for managing variety (namely modularity and product scaling): that of Fujita and coauthors [13], and Hernandez and coauthors [14].
- *Determination of platform extent:* In the majority of top-down techniques, all features or components must be either common to all products or to none of them, typically resulting in dramatic tradeoffs between commonality and performance [10, 15-17]. In order to reduce the impact of commonality on performance, a designer should be able to specify different levels of commonality of the various features and components of the product family.
- *Determination of the number of product variants:* In the beam example described above, it isn't initially clear as to how many product variants should be offered given the complexity of the market demand. Two-thirds of the techniques surveyed by Simpson require specification of the platform *a priori* to optimization. Ideally, the determination of the number of product variants in a platform should be a decision variable that is influenced by both the demand present in the market and the resulting determination of platform extent.
- *Modeling manufacturing costs and market demand:* Half of those techniques surveyed assume that maximizing product performance maximizes demand, maximizing commonality minimizes production costs, and that resolving the tradeoff between the two yields the most profitable product family. Since manufacturing costs and market demand greatly influence decisions relating to platform extent and the number of product variants in the family, these assumptions can lead to sub-optimal product families. Specifically, only half of the methods surveyed integrate manufacturing costs directly, and less than one-third of the methods incorporate market demand or sales into the problem. Of the techniques surveyed that incorporate market modeling in the

formulation of the method, several use traditional market-based analysis to determine the most effective location for a product family in a market space, but do not relate this information to the actual product architecture [18-20]. Those techniques that use a quantitative approach for incorporating customer demand into the formulation of the product architecture [13, 21-23], only model customer demand as uniform across the market. This is not adequate since markets of mass-customized products are characterized as niche and heterogeneous.

As a result of these limitations, designers using existing “optimization-based” top-down approaches are unable to systematically design a satisfactory platform for the cantilever beam example problem, let alone the complex product platforms that are typical of those realized in industry.

## 1.2 Context

In response to these limitations, Hernandez proposes the Product Platform Constructal Theory Method (PPCTM): a novel top-down approach for developing product platforms that facilitates the realization of a stream of customized product variants, and which accommodates the issues of multiple levels of commonality and multiple customizable specifications [24]. The result of the use of the PPCTM is a hierarchical organization of several approaches of commonality, as well as the specification of their range of application across the product platform.

While it has several advantages over current platform design techniques, the PPCTM does have its limitations. Specifically, it is unable to handle the markets of fragmented demand and heterogeneous niches that are inherent in customized products. Also, a designer using the PPCTM is unable to model multiple design objectives. In this paper, the authors present a series of augmentations to the PPCTM in order to address these limitations. Specifically, the authors integrate the utility-based compromise Decision Support Problem and non-uniform demand strategies into the PPCTM. Furthermore, the authors abstract the principles of the PPCTM to apply it in a new domain: the creation of platforms for process parameters.

The PPCTM and its underlying theoretical foundation are described in Section 2. The authors close Section 2 with a discussion of the tools and concepts used in their augmentation of the PPCTM. The augmented method is presented in Section 3. The ability to use the augmented PPCTM for the development of both a product platform (Section 4) and a process parameter platform (Section 5) is shown by revisiting the cantilever beam illustrative example problem. Closing remarks are offered in Section 6.

## 2 AUGMENTING THE PRODUCT PLATFORM CONSTRUCTAL THEORY

### 2.1 The Product Platform Constructal Theory Method

The Product Platform Constructal Theory Method (PPCTM) was developed in order to provide designers a methodical approach for synthesizing multiple methods of offering product variety in the development of product platforms for customized products [14]. As a result of the PPCTM's theoretical foundations in both hierarchical systems theory [25] and constructal theory [26-28], Hernandez represents the design of platforms for customizable products as a problem of optimal access in a geometric space (a detailed description of the theoretical constructs of the PPCTM can be found in [23, 29]).

An optimal access problem is characterized by the need to determine the optimal “bouquet of paths” that link all points of an area,  $S$ , with a common destination,  $O$  (Figure 1a). Bejan proposes to solve access problems through constructal theory, which is centered on a hierarchic process of optimization. A shape that optimizes “access” at the most elementary volume occurs first, followed by an assembly of these innermost “volumes” into a second, larger-shapes, which in turn are assembled into a third volume, and so on [26]. Following the basic tenants of constructal theory, this optimization process should proceed in a specific time direction: from the optimization of the basic elements to the higher-order assemblies of the structure. This sequential process continues until all relevant volume is connected.

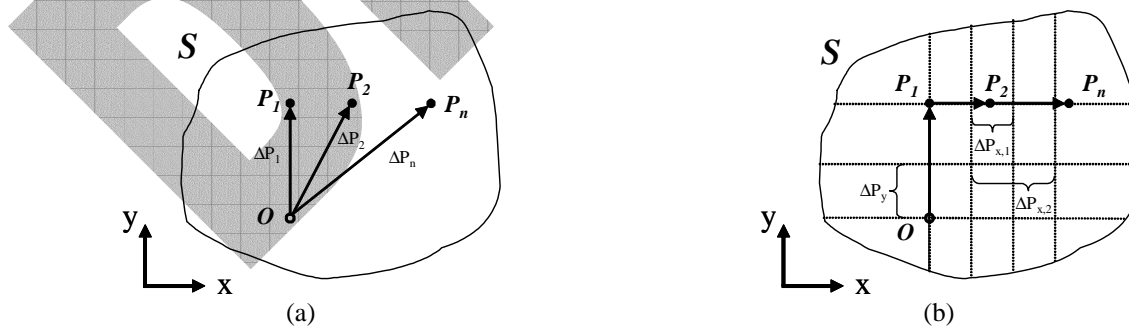


Figure 1. Product Platform Design as a Problem of Optimal Access

In his abstraction of constructal theory and problems of optimal access to product platform development, Hernandez identifies the *space of customization* as the set of all feasible combinations of values of product specifications that a manufacturing enterprise is willing to satisfy (i.e., space  $S$  in Figure 1). Each product

specification for which variety will be offered is represented as a dimension in this space (dimensions  $x$  and  $y$  in Figure 1). The magnitude of each dimension represents the amount of variety that will be offered. Each point in the space of customization represents a variant that the manufacturer wishes to offer.

In the creation of a product family for customized products, a designer wishes to link all different feasible product variants ( $P_1, P_2, \dots P_n$  in Figure 1) within the space of customization from a baseline set of components (the product platform,  $O$  in Figure 1). The manner in which each product variant is linked is through *modes for managing variety* ( $\Delta P_{x,1}, \Delta P_{x,2}, \Delta P_{y,1} \dots \Delta P_{x/y,n}$  in Figure 1b). Modes for managing product variety are any generic approach in product design or its manufacturing process for achieving a product customization (i.e., modular design, platform design and standardization, robust design, dimensional customization, adjustable customization, etc.).

The fundamental problem addressed in the application of the PPCTM to platform design is how to systematically organize and determine the extent of application of these modes for managing product variety across the space of customization. Considering that potential for rapid adaptation is higher in complex systems when they are organized hierarchically, Hernandez proposes to hierarchically organize the multiple of modes for managing product variety. With the modes organized effectively, one must determine their extent of application. Through the application of the tenets of constructal theory, each level of the hierarchy represents a sub-space of the space of customization; the dimensions of each space represent each mode's range of application (Figure 1b).

In order to determine these dimensions, a multi-stage decision is formulated wherein the ranges of application of each mode for managing product variety are the decision variables. The goal of each decision is to find an appropriate compromise among the objective functions (e.g., cost, profit, design performance, etc.) so that an appropriate balance between commonality and performance is achieved. The six steps of the original instantiation of the PPCTM are presented in Figure 2.

## 2.2 Handling Non-Uniform Demand

Simpson states that the inability of product platform design techniques to model the manufacturing costs and the market demand for products in the family can lead to the development of suboptimal product families [12]. As such, both the market demand and the cost of manufacturing have direct influence on the product architecture when designing with the PPCTM. Specifically, the objective function that drives the decision-making process is evaluated as an average over the total number of variants produced (as dictated by the demand). Furthermore, the cost of



manufacturing the entire product family is evaluated by taking the product of the manufacturing cost of each variant and its respective demand.

Unfortunately, the PPCTM example problems presented thus far (pressure vessel [14], beverage merchandiser [24], electric motors [23]) have been solved with the assumption of uniform demand across the space of customization. This approximation does not adequately capture the complexity of a traditional market space for a customized product. Without accurate knowledge of the market demand, a designer is unable to determine the appropriate extent of application of each mode for managing product variety across the platform.

In order to alleviate this limitation, the authors propose two augmentations to the PPCTM. The first step of the PPCTM, “Define the Market Space,” will be expanded to include the development of a model of the demand scenario for the market. The resultant demand model (continuous or discrete) should be expressed in terms of the dimensions of the space of customization. This model will then be carried over into the second step of the PPCTM wherein the objective function is formulated as an average across this varying function of demand. An implementation of this augmentation is illustrated in Section 4.2.

The second proposed augmentation is to eliminate the PPCTM’s explicit constraint that the range of application of each mode for managing variety must be a multiple of the range of application of the mode that supercedes it in the platform hierarchy (i.e., in Figure 1b,  $\Delta P_{x,2} = n\Delta P_{x,1}$ , where  $n$  is an integer). Originally intended to make the design space more tractable, this constraint prevents a designer from truly capturing effect of a non-uniform market demand on the architecture of the product family.

### 2.3 Modeling Multiple Design Objectives

In the second step of the PPCTM (Figure 2), a designer defines an objective function that drives the mathematical optimization of the product platform. In its previous applications, the PPCTM has only shown effectiveness for a single design objective (typically, to minimize the average cost of the product family). This is a cause for concern since multiple, coupled, and conflicting goals become more prevalent as systems are more complex.

In order to provide a designer the ability to handle multiple design objectives, the authors propose to augment the PPCTM by infusing the utility-based compromise Decision Support Problem (u-cDSP). The u-cDSP is a decision support construct that is based on utility theory [30] and permits mathematically rigorous modeling of

designer preferences such that decisions can be guided by expected utility in the context of risk or uncertainty associated with the outcome of a decision [21]. While any appropriate decision formulation technique is serviceable, the authors prefer to formulate each decision stage with the u-cDSP since its use “provides structure and support for including human judgment in engineering decisions involving multiple attributes, while simultaneously providing an axiomatic basis for accurately reflecting the preferences of a designer with regard to feasible tradeoffs among these attributes under conditions of uncertainty” [31]. Furthermore, the u-cDSP has proven useful in previous product platform techniques as it provides a decision construct in which a designer can model multiple, conflicting objectives.

The formulation of each utility-based compromise Decision Support Problem follows the four steps presented in [21]. A utility function for each of the design objectives,  $u(A(X))$ , is formulated by qualitatively and quantitatively assessing the preferences of the designers. These individual utility functions are then combined into a multi-attribute utility function,  $U(X)$ , as a weighted average of the individual utilities. Finally, goal and deviation functions are developed for each decision stage. The deviation function of the u-DSP,  $Z(X)$ , is formulated to minimize deviation from the target expected utility (i.e., 1, the most preferable value), which is mathematically equivalent to maximizing expected utility. The goal and deviation functions formulated for each u-cDSP inherently consider the compromise of the tradeoffs between each objective function. With the goal of minimizing the deviation of the expected utility from the ideal value, parameters that provide the best values for this overall objective are chosen while maintaining consistency with the designer’s preferences.

## **2.4 Offering Manufacturing Capacity Variety through Platform Design**

Aside from making the augmentations mentioned above, the authors strive to further expand the PPCTM by applying it to a new domain: the development of platforms for process parameters for the realization of customized products.

As in the realm of product design, effectively managing modularity and commonality in production process development has been recognized as an important component of producing a large variety of products while maintaining low costs. Realizing that improving the flexibility and productivity of a manufacturing system is the “crucial challenge of modern industrial management” [32], many production process design approaches have been developed to enable manufacturing enterprises to affordably produce customized products.

System-level philosophies such as Cellular Manufacturing [33], Flexible Manufacturing Systems [34], and Reconfigurable Manufacturing Systems [3] focus on reducing setup times, reducing in-process inventory, improving part quality, shortening lead time, improving part quality through grouping similar parts, modules, and components into dedicated cells of manufacturing processes. These philosophies and their related implementation technologies provide general strategic direction for various aspects of the production process – from sequencing and synchronization of multiple machining and assembly operations, to line balancing and capacity planning.

On a lower level of abstraction, Jiao and coauthors introduce the concept of process platforms - a set of similar production processes that share a common process structure - to facilitate coordination in product and process variety management [35]. The resultant design approach aids in the development of the production process plan and structure by taking advantage of the common production processes required to manufacture all of the product variants.

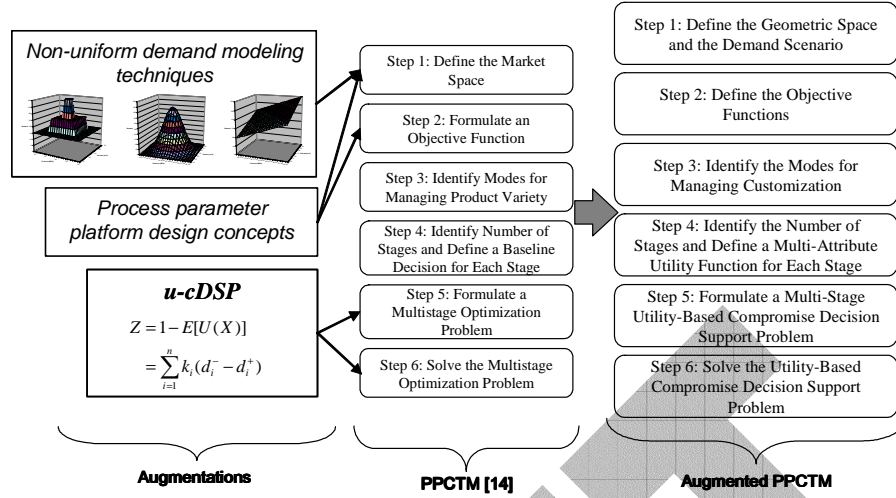
It is the authors' assertion that platform design techniques are applicable at an even lower level of abstraction in the problem of manufacturing process design. In the context of a single workstation of a production process, frequent changes in production capacity requirements force a manufacturing engineer to reconfigure its process parameters (e.g., turning speed, tool size, laser power, operating temperature, etc.) in order to maintain the best compromise between conflicting process objectives (e.g., minimization of cost, maximization of throughput, maximization of quality). Such reconfiguration requires re-evaluation of the process parameters, and entails a costly and lengthy setup of the workstation. In this context, the core concept of platform design – offering variety efficiently through commonality and/or modularity – can be applied to reduce workstation setup penalties. As such, the concept of a *process parameter platform* is introduced:

*A process parameter platform* is defined as a set of common process parameters from which a stream of derivative process parameters can generate a customized machining process efficiently despite changes in required capacity.

The crux of process parameter platform design is the commonalization of process parameters such that transitions between different workstation setups are handled efficiently and effectively. The application of the augmented PPCTM to this new domain is illustrated in Section 5.

### 3 THE AUGMENTED PRODUCT PLATFORM CONSTRUCTAL THEORY METHOD

The augmentation of the PPCTM, as described in the previous section, is presented graphically in Figure 2.



**Figure 2.** Augmenting the Product Platform Constructral Theory Method

The first step of the augmented PPCTM involves the space of customization through (i) the identification of the design specifications that will be varied according to the customer demands (the dimension of the space), (ii) the range of variety that will be offered for each specification (the bounds of the space), and (iii) the analysis and modeling of the demand of the market. The choice for best model for demand of a market is context based. Although not explicitly illustrated in this paper, the reader should be assured that this methodology is robust to any non-uniform demand model that a designer may choose, as shown in [36].

For the development of a process parameter platform, there is but one design specification in which a manufacturer wishes to offer variety: production capacity. Thus the space of customization for process parameter platform design is a single dimension, bounded by the range of production capacity that the manufacturing enterprise wishes to offer.

In the second step, objective functions are defined. In order to handle the tradeoff between platform extent and performance, the objective functions are evaluated as averages across the space of customization. Typical objective functions for product platform design include the minimization of cost, or the improvement of a product performance metric. For process platform design, objective functions can include minimization of cost, maximization of throughput, and maximization of part quality among others.

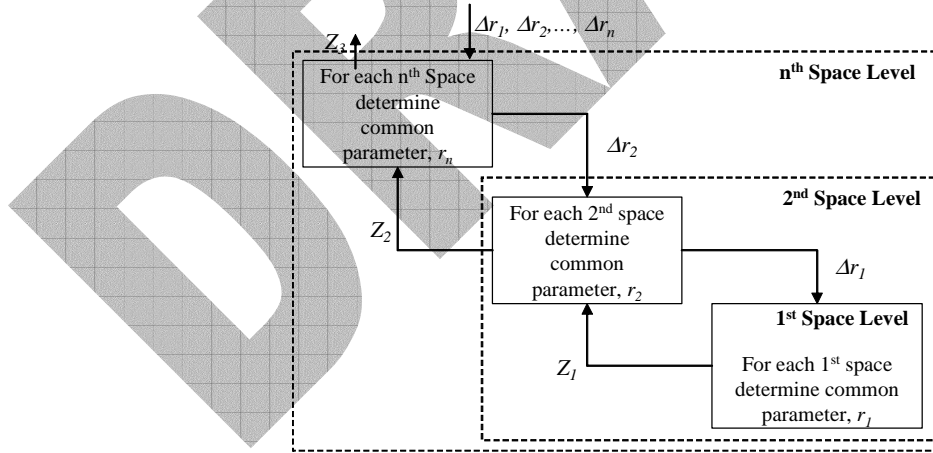
The modes for managing variety (defined in Section 2.1) are identified in the third step. These modes for managing variety are the linking mechanism between the variants that compose the product family. Graphically, they represent a geometric “sub-space” of the entire space of customization; the size of each sub-space represents

the extent of application of each mode. The determination of these modes is a strategic decision that involves decision-making in the context of both design and manufacturing.

The modes are then hierarchically organized in Step 4. Modes that are capable of achieving the smallest variations in the varied design parameters are typically used at the lower levels of the hierarchy (i.e., before modes that can only achieve large variations in the design parameters). Economical and technological considerations place an important role in establishing the hierarchic use of the modes for managing variety.

The determination of the range of application of each mode for managing variety is accomplished through the formulation and solution of a multi-stage utility-based compromise Decision Support Problem (Steps 5 and 6). Following the tenets of constructual theory, the determination of the range of application of each mode for managing variety that composes a level of the hierarchy represents one stage in a multi-stage decision. While any appropriate decision formulation technique is serviceable, the authors prefer to formulate each decision stage with a utility-based compromise Decision Support Problem (u-cDSP; Section 2.3).

The design decisions of each stage are coupled with one another; therefore, in previous work, methods such as dynamic programming [24], linear physical programming, and the construction of response surfaces have been used to decouple these decisions. In this paper, the authors have elected to implement an exhaustive search algorithm (Figure 3).



**Figure 3.** Exhaustive Search Solution Algorithm for Multi-Stage PPCTM

This solution method involves iterating through values of the modes for managing variety ( $\Delta r_1, \Delta r_2, \dots, \Delta r_n$ ), establishing the dimensions of the sub-spaces, commonalizing the values of the design parameters ( $r_1, r_2, \dots, r_n$ ) across each sub-space, evaluating the objective functions, and comparing the resulting overall utility of each

iteration. With the extent of application of each mode known, a designer is capable of fully defining the platform architecture that offers the best compromise to the objective functions.

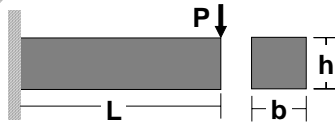
This augmented method is illustrated through its application to the example problem proposed in Section 1.1. The augmented PPCTM is used to design a product platform for a line of customizable cantilever beams in Section 4. The augmented PPCTM is then extended through its usage in designing a process parameter platform for the manufacture of a line of customizable cantilever beams in Section 5.

#### 4 PPCTM APPLICATION: PRODUCT PLATFORM DESIGN

In this section the authors illustrate the design of a product platform using the PPCTM. It should be noted that the example is kept fairly simple in order to focus the reader's attention on the method itself. It is important to keep in mind that the example's emphasis is on illustrating the method rather than the results *per se*. It is assumed that uncertainty and risk are absent from this problem. It is also noted that some values used in the example are estimates and do not change the fundamental results of this paper. The model of the cantilever beam can easily be modified to suit specific situations; however, the authors' focus is centered on the validation of the method itself.

##### 4.1 Example Problem: Customizable Cantilever Beams

Revisiting the example posed in Section 1.1, we consider the following scenario: a manufacturer wishes to offer a line of customizable cantilever beams (Figure 4). The manufacturer wishes to provide customers the ability to specify a beam that ranges in length ( $L$ ) from 0.5 to 10 m, and is capable of supporting a single end-load ( $P$ ) from 50 to 500 N.

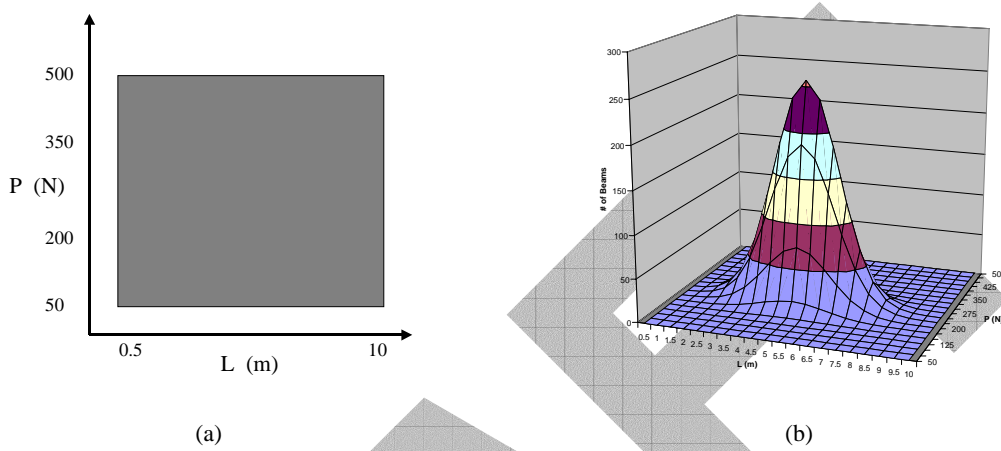


**Figure 4.** Schematic of Cantilever Beam Embodiment

The manufacturer has decided to offer variety via three methods, (i) by changing the beam cross-section, (ii) by changing the beam material, and (iii) by cutting the beams to customized lengths from standard pieces. The manufacturer has two conflicting goals: to provide the lowest average cost across the volatile market and to provide the lowest average maximum beam deflection across the family (i.e., to improve product quality).

## 4.2 Step 1: Define the Geometric Space and Demand Scenario

For the design of the family of customizable beams, there are two independent design specifications that characterize the desired product customization - the beam length and the applied load. The resulting two-dimensional continuous space of customization illustrated in Figure 5a.



**Figure 5.** Space of Product Customization for Cantilever Beam Example

For this problem, the manufacturer has observed that there is significantly more demand for the medium-ranged cantilever beams. More specifically, the manufacturer has determined that the most appropriate manner of modeling demand is to treat it as a normal distribution (Figure 5b), as expressed generally in Equation 1,

$$D(x, y, \dots, n) = \prod_i^n \left( A_i \frac{1}{\sigma_i \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x_i - \mu_i}{\sigma_i} \right)^2} \right) \quad (1)$$

where  $\mu_i$  is the mean and  $\sigma_i$  is the standard deviation of each dimension in the  $n$ -dimensional market space ( $\mu_l = 5\text{m}$ ;  $\mu_p = 250\text{N}$ ). The variable  $A$  is a scaling coefficient that translates the probability of a product being ordered to the number of products being ordered ( $A = 250$  products).

## 4.3 Step 2: Define the Objective Functions

For the design of the product platform for this example, the manufacturer has two objectives: to minimize the average product cost and to minimize average deflection of the products in the entire market space.

The average total cost of the product family is calculated by averaging the cost of each individual product variant over the entire market as seen in Equation 2,

$$C_{product,avg} = \frac{1}{D_{tot}} \left[ \sum_{i=1}^n D_i (C_{material,i} + C_{order,i} + C_{waste,i}) \right] \quad (2)$$

where  $n$  is the total number of product variants,  $D_i$  is the demand of the specific product variant  $i$ ,  $D_{tot}$  is the total demand of the all of the products of the market space (the summation of Equation 1 across all variants),  $C_{material}$  is the material cost,  $C_{order}$  is the cost required to order the material, and  $C_{waste}$  is the cost associated with the material wasted from cutting beams to length from standard lengths.

The average deflection of the beams of the product family is found by summing the deflection of each product variant, and then averaging that sum over the entire market, as shown in Equation 3,

$$w_{avg} = \frac{1}{n} \sum_{i=1}^n \frac{-PL_i^3}{3E_i I_i} \quad (3)$$

where  $P$ ,  $L$ ,  $E$ , and  $I$  are the load, length, modulus of elasticity, and moment of inertia of each beam, respectively.

#### 4.4 Step 3: Identify the Modes for Managing Variety

For the cantilever beam example, a designer must identify methods in which to offer product variety in beam length and beam strength. After consulting with manufacturing, and evaluating the firm's core competencies, designers have chosen three modes of managing product variety - two for offering variety in strength, and one for offering variety in length:

- *Mode  $P_1$  - Standardization of Beam Cross-Section:* Variety in beam strength can be offered by changing the cross-section of the beam. The manufacturer has decided to keep the beams' width ( $b$ ) constant at 0.15 m since the height of the beam,  $h$ , has a more direct affect on the beam's rigidity. Therefore, for this mode, the beams' height is varied from 0.1 m to 0.25 m.
- *Mode  $P_2$  - Standardization of Beam Material:* This mode is used to achieve changes in beam strength by choosing one of three available materials: fir, pine, and hickory. Each material differs in modulus of elasticity, ultimate stress, and cost. The properties of each of these materials are presented in Table 1. The result of the application of this mode is the determination of the range of application of each material across the product family.
- *Mode  $L_1$  - Dimensional Customization of the Beam Length:* For this mode, changes in length are achieved by cutting each beam from a stock piece of raw material, while maintaining a constant cross-section. The result



of this mode is the use of a common cross-section and a common stock length of material for a certain range beam lengths.

These three modes (Modes  $P_1$ ,  $P_2$ ,  $L_1$ ) are the approaches considered for accessing all the points of the space of customization (Figure 5).

The application of each of these modes presents a tradeoff for the manufacturer. For example, while the standardization of the beam cross-section will reduce order costs and improve the average deflection of the beam family, it will greatly increase the material cost from over-designing the lower-end beams.

**Table 1.** Material Properties for Cantilever Beam Example

<i>Material</i>	<i>Elasticity (MPa)</i>	<i>Ultimate Stress (GPa)</i>	<i>Cost (\$/m<sup>3</sup>)</i>
Hickory	15	63	35
Fir	13	50	25
Pine	9	36	15

#### 4.5 Step 4: Identify the Number of Hierarchy Levels and Allocate the Modes for Managing Variety to the Levels

In the fourth step, it is established how and when each mode of managing variety is used.

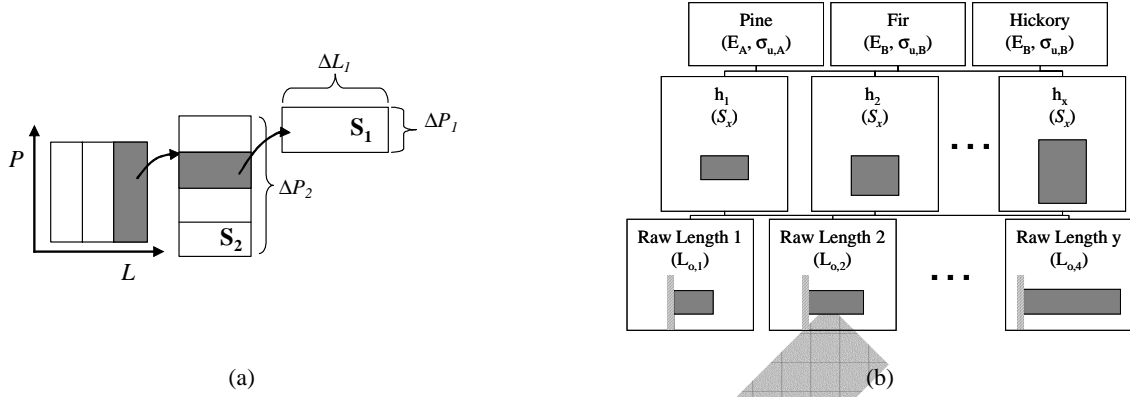
##### The First Stage and the First Space Element

For the first space element, modes  $L_1$  (dimensional customization of beam length) and  $P_1$  (standardization of beam cross section) are used in concert to achieve variety in both length and strength of the beam. These modes are the cheapest means of offering extremely small changes between product variants in both customizable specifications.

The size and shape of this first space element,  $S_1$ , is given by the value of the variables  $\Delta L_1$  and  $\Delta P_1$  as shown in Figure 6a.  $\Delta L_1$  and  $\Delta P_1$  represent fundamentally the extent to which the raw length and the beam cross-section are commonalized across the market respectively.  $\Delta L_1$  and  $\Delta P_1$  are therefore decision variables for this first stage.

##### The Second Stage and the Second Space Element

The second space element,  $S_2$ , is composed by a number of assemblies of the first space element,  $S_1$ , in the load dimension through the application of Mode  $P_2$ , “Standardization of Beam Material.” The size and shape of these space elements is given by the value of the variables  $\Delta P_2$  and  $\Delta L_1$ , as shown in Figure 6a.  $\Delta P_2$  represents the extent to which the beam material is commonalized across the market. A graphical interpretation of the hierarchic synthesis of the modes for managing product variety is provided in Figure 6b.



**Figure 6.** Graphical Representation of the Organization of the Modes for Managing Product Variety

#### 4.6 Step 5 & 6: Formulate and Solve the Multi-Stage Utility-Based Compromise Decision Support Problem

The focus in the first decision stage of the cantilever beam example is the determination of the range of application of Mode  $L_1$  (dimensional customization of beam length,  $\Delta L_I$ ) and Mode  $P_1$  (standardization of beam cross-section,  $\Delta P_I$ ) that will minimize the average cost (Equation 2) and the average deflection (Equation 3) of the subspace. Following the steps outlined in Section 2.3, individual utility functions are created for each of the objectives,  $u_i(A_i(X))$ , and then combined into a multi-attribute utility function,  $U(X)$ , as a weighted average:

$$U = k_c u_c + k_w u_w \quad (4)$$

where

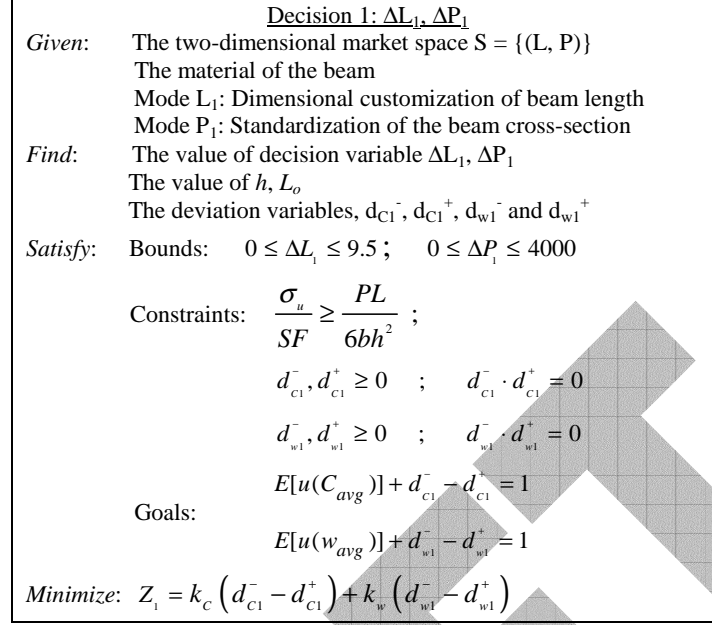
$$u(C_{avg}) = 0.0001C^2 - 0.0203C + 0.966 \quad (5)$$

$$u(w_{avg}) = 30.68w^2 + 10.79w + 0.9284 \quad (6)$$

and,  $k_w$  and  $k_c$ , are the scaling constants (determined by evaluating a designer's tradeoff preferences;  $k_c = 0.55$  and  $k_w = 0.45$ ).

Finally, goal and deviation functions are developed for each stage. The deviation function of the u-DSP,  $Z(X)$ , is formulated to minimize deviation from the target expected utility of 1 (Section 2.3). The deviation function of the cantilever beam example is:

$$Z = 1 - (E[U(C)] + E[U(w)]) = k_c (d_c^- - d_c^+) + k_w (d_w^- - d_w^+) \quad (7)$$



**Figure 7.** Formulation of the Multi-Stage Compromise Decision Support Problem for the Cantilever Beam, First Stage Element

The resulting decision formulation for this first stage of the cantilever beam example is presented in Figure 7. As can be seen in Figure 7, the decision variables for each stage are the extent of application of each mode of managing variety ( $\Delta L_1, \Delta P_1$ ), as well as the value of the design variables that are being commonalized across the subspace (the beam height,  $h$ , and the stock length from which the beam will be cut,  $L_o$ ). It can also be seen from Figure 7 that engineering constraints are incorporated into the formulation of each u-cDSP. For the product platform design of the cantilever beam example problem, the sole engineering constraint is that the maximum stress of the beam, with its specified length, material, cross section, and applied load, is not greater than the material's allowable stress (including a consideration of an appropriate safety factor).

In order to maintain brevity, the remainder of the stages' decision formulations will not be presented. The reader should be confident that a similar process to that shown above is used in creating the u-cDSP for the second stage of the product platform multistage decision.

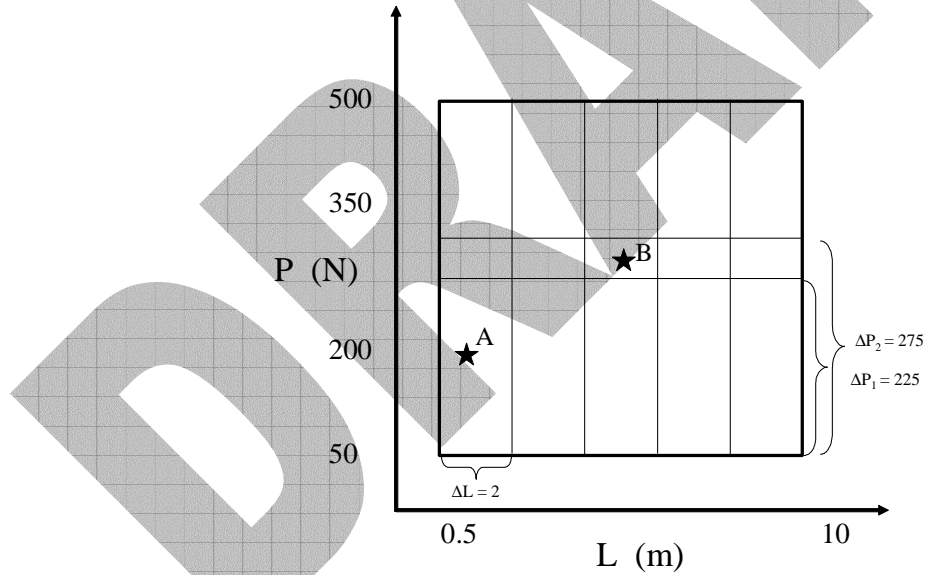
It should be noted that in this example problem risk and uncertainty are not modeled. Although this feature of the u-cDSP is not exercised, the u-cDSP is employed for its ability to quantify a designer's preference in the presence of multiple objectives. The formulation of the u-cDSP is done in order to maintain connectivity to the authors' previous work and to provide readers the confidence that the method is capable of handling a more complex set of problems.

## 4.7 Results

The multi-stage u-cDSP formulated in the Section 4.6 is solved via exhaustive search as outlined in Section 3 (Figure 3). The result of the application of the augmented PPCTM to this example problem is the hierarchic organization of the multiple modes for managing variety, and the determination of their range of application across the product platform that provides the best compromise the conflicting design objectives (Table 2). The results inform the manufacturing enterprise that the best configuration of the modes of managing variety to minimize cost and deflection is to commonalize the raw length for every 2 m of beam length, commonalize the beam cross section for every 225 N of loading, and to commonalize the beam material for every 275 N. This space of customization is presented in Figure 8.

**Table 2.** Range of Application of the Modes for Managing Product Variety

$\Delta L_1$ (m) (customization of length)	$\Delta P_1$ (N) (standardization of cross-section)	$\Delta P_2$ (N) (standardization of material)
2.0	225	275



**Figure 8.** Space of Customization for Cantilever Beam Space of Customization

From these ranges of application for each mode of managing variety, the specific values of the design variables are derived by the use of the PPCTM (Table 3).

**Table 3.** Mapping Between Product Specifications and Design Variables

L	P	h	Material
2.5	275	0.235	Pine
	325	0.25	Pine
	500		
4.5	275	0.25	Pine
	325	0.25	Pine
	500		
6.5	275	0.25	Pine
	325	0.25	Fir
	500		
8.5	275	0.25	Fir
	325	0.25	Fir
	500		
10	275	0.25	Fir
	325	0.25	Fir
	500		

The results presented in Table 3 serve as a “roadmap” for a designer; any potential combination of beam length and beam strength for the considered space of customization is connected to specific design parameters, including dimensions of raw material. For example, if a customer requests a beam of 1 m that can support a load of 200N (point A in Figure 8), the manufacturer would cut a 2.5m raw pine beam of cross section 0.15m x 0.235m to 1m. If another customer requests a 5.75m beam that can support a load of 315N (point B in Figure 8), the manufacturer would cut 0.75m off the length of a 0.15m x 0.25m x 6.5m pine beam.

To further demonstrate the ability of the method to incorporate non-uniform demand, the example problem is re-solved with multiple demand scenarios. Two additional demand scenarios are presented (normal distributions in the high-end and low-ends of the market) in Table 4 along with the resulting range for applying each mode for managing variety. As can be seen from the results, the architecture of the product family is dependent on the demand scenario of the space of customization.

**Table 4.** Range of Application of the Modes for Managing Product Variety for Multiple Demand Scenarios

<i>Scenario</i>	<b>Mean, L (m)</b>	<b>Mean, P (N)</b>	<b>L<sub>1</sub></b>	<b>P<sub>1</sub></b>	<b>P<sub>2</sub></b>
<b>Normal, center</b>	5	250	2	225	275
<b>Normal, high</b>	7.25	350	2	300	300
<b>Normal, low</b>	2.5	175	1.5	450	450

Comparing the results of the normal distribution of demand in the center of the market with that of the lower-end of the market shows that the range of material and cross-section commonality has dramatically increased while the stock length commonality has slightly decreased. This is to be expected; the penalties that would typically occur from commonality of material and cross-section (increase in material costs) are outweighed by the benefits (reduction in average beam deflection) due to the lack of demand for high-end products.

The PPCTM's ability to directly accommodate for the tradeoffs between platform extent and product performance in the face of multiple modes of managing variety for multiple varied design specifications and non-uniform demand corroborates previous claims of the effectiveness of the method itself [36].

## **5 PPCTM APPLICATION: PROCESS PARAMETER PLATFORM**

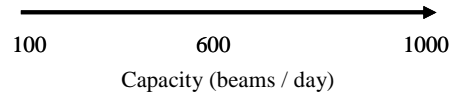
In this section the authors illustrate the design of a process parameter platform using the PPCTM (described in Section 2.4). In order to maintain continuity, the example concerns the manufacture of customizable cantilever beams. Similar to its product platform counterpart, the cantilever beam example of this section may be considered simple, but it is not trivial: the problem requires that a designer handle multiple design objectives, multiple modes of managing variety, variety in multiple design parameters, and the tradeoff between commonality and performance.

### **5.1 Example Problem: Manufacturing Customizable Cantilever Beams**

Consider the following scenario: a manufacturer of customized cantilever beams is concerned with maintaining the efficiency of a workstation of the production process, since the capacity requirements of the make-to-order process significantly fluctuate between 100 to 1000 beams per day. As the capacity requirement changes, the manufacturing engineer is required to re-evaluate and re-configure the workstation's process parameters: the speed at which the material is cut, the size of each production batch, and the number of and type of machines to be used. The cost and time penalties associated with each new setup conflict with the three design objectives for the process: the minimization of cost, the maximization of throughput, and the maximization of part quality. The problem is further obfuscated by a week-long lead time production constraint.

### **5.2 Step 1: Define the Geometric Space and Demand Scenario**

The space of customization for process parameter platform design is a single dimension, bounded by the range of production capacity that the manufacturing enterprise wishes to offer. The resulting space of customization is shown in Figure 9. Each point along this space of customization represents a different level of production capacity.



**Figure 9.** Cantilever Beam Example Space of Production Capacity Customization

### 5.3 Step 2: Define the Objective Functions

As stated in Section 5.1, the two objectives in the development of the platform are the minimization of the cost, the maximization of throughput, and the maximization of part quality. The average cost of manufacturing the cantilever beam would incorporate machine costs, operation costs, maintenance costs, and material costs. In order to capture the benefit of developing a process parameter platform, models for the cost and throughput of the production process need to incorporate the penalties incurred due to the changeover between different process parameters.

### 5.4 Step 3: Identify the Modes for Managing Variety

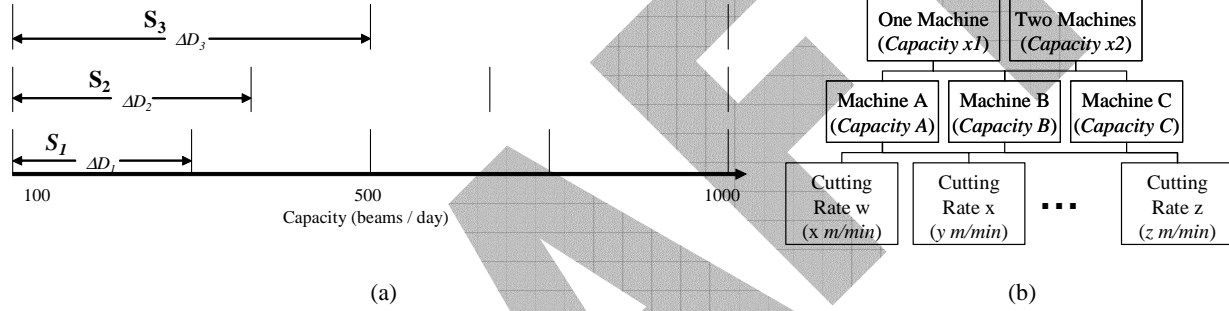
For the development of a process platform for the cantilever beam example, the manufacturer has chosen the following modes for managing variety:

- *Mode  $D_1$  - Standardization of Cutting Rate:* The rate at which the beams are cut has a large influence on the production rate of the manufacturing process. High fidelity changes in production capacity can be made through control of the cutting rate.
- *Mode  $D_2$  - Standardization of Machine Type:* There are several types of machines available for the production process. Different machine types offer variety in batch sizes and production rate. This mode offers large, discrete changes in production capacity.
- *Mode  $D_3$  - Modular Combination of Machines:* Adding another machine to the manufacturing system provides a very large increase in production capacity. While much greater production capacity can be

achieved through the implementation of this mode, the manufacturer must take into account the significant increase in cost.

Similar to the design of the product platform, these modes of managing variety present several tradeoffs. For example, while standardizing the cutting rate would minimize setup penalties, it may also decrease part quality and increase production costs.

#### 5.5 Step 4: Identify the Number of Hierarchy Levels and Allocate the Modes for Managing Variety to the Levels



**Figure 10.** Hierarchic Organization of the Modes for Managing Production Customization for the Cantilever Beam

##### The First Stage and the First Space Element

For the first space element, Mode  $D_1$  (standardization of the cutting rate) is used to achieve variety in production capacity. Of the three modes of offering variety, standardizing the cutting rate provides the manufacturer the greatest control over production capacity; thus this mode is placed at the lowest level of the hierarchy. The size and shape of this first space element,  $S_1$ , is given by the value of the variable  $\Delta D_1$  as shown in Figure 10a.

##### The Second and Third Stages and Space Elements

The second space element,  $S_2$ , is composed by a number of assemblies of the first space element,  $S_1$ , as shown in Figure 10a. The size of this space element is given by the value of the variable  $\Delta D_2$ , the standardization of the machine type. The third space element,  $S_3$ , is composed by a number of assemblies of the second space element,  $S_2$ . The size of this element is given by the value of variable  $\Delta D_3$ , the modular combination of machines.

Although these are two distinct space elements, their implementation is very similar. Both modes are based on the concept that certain capacity requirements are suited for different types and quantities of machines. Whether



limited by maximum capacity or by a cutting speed, lower-end machines simply cannot produce parts at a sufficient rate to meet larger demands. This applies to the other end of the capacity space spectrum as well; higher-end machines may be too expensive to justify their use for lower capacity needs. Since these two modes are only able to offer large, expensive, discrete changes in production capacity, they are used at the highest level of the hierarchy. Obviously, a machine setup of a certain type and quantity that can satisfy a capacity  $D_2$  will also satisfy any capacity  $D_1$ , where  $D_1 \leq D_2$ . Therefore, the focus in this decision stage is the assignment of different machine types and quantities to specific ranges of the capacity space.

A graphical interpretation of the hierarchic synthesis of the modes for managing production capacity variety is provided in Figure 10b.

## 5.6 Steps 5 & 6: Formulation and Solution of the Multi-Stage Utility-Based Compromised Decision Support Problem

In order to maintain brevity, the specific decision formulations of this example problem are not presented. Following the same approach found in the design of a product platform (Sections 4.6 and 4.7), the determination of the range of application for each mode for managing capacity variety that composes a level of the hierarchy (or subspace) represents one stage in a multi-stage decision. These coupled decisions are formulated as a u-CDSP wherein the extent of application of each mode of managing variety are the key decision variables.

## 5.7 Results

The extent of application of each mode of managing production variety for the cantilever beam's process parameter platform is shown in Table 4.

**Table 4.** Range of Application of the Modes of Managing Production Variety

$\Delta D_1$ (beams/day) (standardization of cutting rate)	$\Delta D_2$ (beams/day) (standardization of machine type)	$\Delta D_3$ (beams/day) (combination of machines)
165	450	750

From this solution, the manufacturer knows that the cutting speed should be standardized for every 165 beams/day, the type of machine should be commonalized every 450 beams/day, and that for capacities greater than 850 parts/day, the process should use two machines. Similar to the development of the product platform, these ranges of application for each mode of managing variety correlate to specific values of the design variables (Table 5). For

example, if the capacity requirement for the week is determined to be 445 beams, the manufacturer knows that one Machine B should be used at a cutting rate of 0.5 m/min.

**Table 5.** Mapping Between Production Platform and Process Parameters

Capacity (beams/day)	Cutting Rate (m/min)	Machine Type	Number of Machines
100	0.1	A	1
265	0.22	A	1
430	0.48	A	1
550	0.5	B	1
595	0.5	B	1
760	0.54	B	1
850	0.63	B	2
925	0.63	B	2
1000	0.7	C	2

## 6 CLOSURE

In this paper the authors present augmentations for the Product Platform Constructal Theory Method (Section 2) that enable a designer to systematically manage modularity and commonality in the development of platforms in the presence of multiple design objectives and non-uniform demand in the market. Although developed primarily for the realization of product platforms, the authors have taken advantage of the PPCTM's ability to strategically manage commonality in order to expand its application to the domain of process parameter design. By abstracting the concepts of product platform design, process parameters are systematically commonalized in order to make workstation setup transitions more efficient.

An illustrative problem, the design of a product platform (Section 4) and a process parameter platform (Section 5) for the realization of a line of customizable cantilever beams, is presented to aid in the description of the methodology in both domains. Despite its simple façade, the example problem is not trivial as it requires a platform design method that is capable of handling multiple design objectives, synthesizing multiple techniques for managing variety for multiple design specifications, modeling manufacturing costs and market demand, and explicitly determining the platform extent and the number of product variants (Section 1.1). From its solution it is shown that the use of the augmented PPCTM provides a designer the ability to accommodate the issues of:

- *multiple design objectives*: As described in Sections 4.3 and 5.3, the development of both platforms required the ability to handle multiple, conflicting objectives (i.e., minimization of cost and minimization of deflection for

the product platform, and the minimization of cost, maximization of throughput, and maximization of part quality for the process parameter platform).

- *multiple modes of offering variety for multiple design specifications*: As described in Sections 4.5 and 5.5, the PPCTM synthesizes multiple modes of offering variety through hierarchic organization. These means of achieving all variants within the space of customization are organized in order to offer variety efficiently.
- *volatile markets*: As shown in Sections 4.2 and 5.2, the definition of the space of customization allows the designer to incorporate the complex nature of the non-uniform market demand for the development of both platforms.
- *inherent tradeoffs between platform extent and performance*: As shown in Sections 4.6 and 5.6, the range of application of each mode for managing variety is systematically determined through the rigorous formulation of utility-based compromise Decision Support Problems.

In the context of Simpson's review, the augmented PPCTM is the only optimization-based platform design approach that provides a designer the ability to develop platforms in the presence of all of these problem characteristics.

Opportunities for future work with the PPCTM have been identified:

- *handling risk and uncertainty*: Although the u-cDSP decision formulation is imbedded in the augmented PPCTM, the example problem solved in this paper assumes an absence of uncertainty. In order to account for uncertainty, a designer would have to develop a probability distribution function for those variables (or models) that are uncertain. In order to solve the u-cDSP, appropriate statistical methods (e.g., Monte Carlo simulation) would be used to evaluate the deviation function in terms of the distribution function. The expected utility is then calculated as the mean of the resultant range of solutions.
- *Determining which variables to commonalize and the means to do so*: In its current state, the selection and the organization of the modes of managing variety in the PPCTM are not guided by a systematic process. This is also true for the selection of the variables that are the most advantageous to commonalize.
- *Exploring non-uniform application of each mode for managing variety*: In order to handle non-uniform demand, the authors relaxed an existing constraint in the PPCTM that forced the range of application of each mode for managing variety to be a multiple of the modes that supersede it in the hierarchy. This constraint was an unnecessary relic of the abstraction of constructal theory to platform design. It can be argued, however, that to provide the flexibility necessary to handle non-uniform demand, the extent of application of

each mode should not be required to be uniformly distributed across the market space. Removing this constraint, however, would add significant complexity to the solution of the PPCTM.

- *Handling an evolving market:* A designer using the PPCTM is forced to define a fixed space of customization.

This assumption prevents the development of product platforms that can adapt to future changes in the market place. As such, we are currently investigating the use of constructal theory as a tool to explore issues of strategic product redesign.

## ACKNOWLEDGMENTS

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